**Frontal Advance Theory**

Saturations below the bubblepoint

- Below the bubblepoint, a free gas saturation exists in the pore spaces.
- As the reservoir pressure increases to a level above the bubble point, gas will go back into solution with the oil.
- The more depleted the reservoir, the longer the time to fill-up, and therefore, the longer the time to waterflood response.

**Fractional Flow Equation**

We have to study the front....

\[
J_w = \frac{q_w}{q_w + q_o}
\]

\[
WOR = \frac{q_w}{q_o}
\]

\[
q_t = q_w + q_o
\]
Fractional Flow Equation

The fractional flow equation is a model used to determine the water fraction of the total fluid flow at a particular location and time in a linear reservoir waterflood.

- It determines the location and time for a fractional flow: distances against the saturation (front) after specific date:

\[
q_{W} = \frac{A \cdot k_{W}}{\mu_{W}} \left( \frac{\partial P_{W}}{\partial L} - 0.433 \gamma \sin \alpha \right)
\]

\[
q_{o} = \frac{A \cdot k_{o}}{\mu_{o}} \left( \frac{\partial P_{o}}{\partial L} - 0.433 \gamma \sin \alpha \right)
\]

\[
P_{c} = P_{W} - P_{o} = \frac{\partial P_{c}}{\partial L} = \frac{\partial P_{W}}{\partial L} - \frac{\partial P_{o}}{\partial L}
\]

Dip angle

Capillary pressure term
(usually ignored)

\[
1.127 \times 10^{-3} A \kappa_{o} \frac{\partial P_{c}}{q_{W} + q_{W}}
\]

Gravity term

\[
1.127 \times 10^{-3} A \kappa_{o} \left( 0.433 \gamma \sin \alpha \right)
\]

\[
\frac{1 + \frac{1.127 \times 10^{-3} A \kappa_{o} \left( \frac{\partial P_{c}}{q_{W} + q_{W}} - 0.433 \gamma \sin \alpha \right)}{1 + \frac{\mu_{w}}{\mu_{o} k_{w}}}}
\]

The actual magnitude of capillary forces and \( \gamma \sin \alpha \) is small and difficult to accurately determine, if not impossible to evaluate; therefore, it is usually omitted from the equation.
Analyzing Waterflood Patterns

**Fractional Flow Equation**

**Horizontal reservoir**

\[ f_w = \frac{1}{1 + \frac{\mu_w}{\mu_o} \frac{k_{ro}}{k_{rw}}} \]

Please note that \( k_{ro} = a c^{n_r} \)

\[ f_o = \frac{1}{1 + \left( \frac{\mu_w}{\mu_o} \right) a c^{n_o}} \]

\[ \frac{df_w}{dS_w} = \frac{\left( \frac{\mu_w}{\mu_o} \right) a c^{n_r}}{1 + \left( \frac{\mu_w}{\mu_o} \right) a c^{n_o}} \]

**Fractional Flow of Water**

- If the reservoir is horizontal, the fractional flow equation is simplified because we can ignore gravity and capillary pressure.
- This equation is evaluated for a point in the reservoir at a point in time. This explicitly defines a water saturation.
- Taking the water saturation and entering the relative permeability curves provides \( k_{ro} \) and \( k_{rw} \).
- Oil and water viscosities for the average reservoir pressure are obtained from laboratory data or correlations.

**Fractional Flow Equation**

- \( F_w \) is function of the saturation...\( Ko/kw \)
- Viscosities ratio are almost constant
- In the fractional flow equation, the ratio of the relative permeabilities in the equation is the ratio at a given/specific saturation—that is, at one point in the reservoir.
- However, in the mobility ratio equation, the water permeability is that in the water-contacted portion of the reservoir, and the oil permeability is that in the oil bank—that is, at two different and separated points in the reservoir. (End points)

**Fractional Flow of Water is Affected by:**

<table>
<thead>
<tr>
<th>Increased Value of Term</th>
<th>Effect on Fractional Flow of Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>injection rate</td>
<td>increase</td>
</tr>
<tr>
<td>capillary pressure gradient</td>
<td>increase</td>
</tr>
<tr>
<td>permeability to oil</td>
<td>decrease</td>
</tr>
<tr>
<td>( k_o/k_w )</td>
<td>decrease</td>
</tr>
<tr>
<td>cross sectional area</td>
<td>decrease</td>
</tr>
<tr>
<td>( \mu_w/\mu_o )</td>
<td>decrease</td>
</tr>
<tr>
<td>fluid density difference</td>
<td>decrease</td>
</tr>
<tr>
<td>dip angle</td>
<td>decrease</td>
</tr>
</tbody>
</table>
Several important pieces of information can be derived from the fractional flow curve. By drawing a straight line tangent to the fractional flow curve, starting at $f_w = 0$ and $S_w = S_{wi}$,

1) At the tangent point, the corresponding $S_w$ is the water saturation at the flood front.
2) The corresponding $f_w$ is the fraction of water flowing at the flood front.
3) The water saturation value where the tangent line intersects ($f_w = 1.0$) is the average water saturation in the reservoir at breakthrough.

Note: This is for a single-layer system.
4) Displacement efficiency ($E_D$) at breakthrough is calculated from:

$$E_D = \frac{S_{wbt} - S_{wi}}{1 - S_{wi}}$$
Example 2: Fractional Flow Curve

Fractional Flow Curve

1. $S_w = 55\%$
2. $f_w = 82.5\%$
3. $\bar{S}_{wBT} = 63\%$
4. $E_D = \frac{0.63 - 0.2}{1 - 0.2} = 0.5375$

Mobility

\[
\text{mobility} = \frac{\text{permeability of rock to fluid}}{\text{fluid viscosity}}
\]
Significance of Mobility Ratio

- Mobility, as taken from Darcy's equation is the permeability of the rock to that fluid divided by the viscosity of the fluid.
  
  Water mobility is \( \frac{k_w}{\mu_w} \)
  
  Oil mobility is \( \frac{k_o}{\mu_o} \)
  
- Mobility is a function of saturation.

Mobility Ratio

\[
M = \frac{\text{Mobility of Water}}{\text{Mobility of Oil}} = \frac{k k_{rw}}{k k_{ro}} = \frac{\mu_w}{k_{rw}} \frac{\mu_o}{k_{ro}} = \frac{k_{rw} * \mu_o}{k_{ro} * \mu_w}
\]

Water relative permeability is taken at average water saturation behind the flood front while oil relative permeability is taken at oil saturation ahead of the front. Mobility ratio calculated this way is sometimes referred to as end-point mobility ratio.

Significance of Mobility Ratio

- Mobility ratio is defined as a ratio of the mobility of the displacing fluid to the displaced fluid.

\[
M = \frac{\lambda_{DP}}{\lambda_{D}} = \frac{\left( \frac{k}{\mu} \right) \text{displacing}}{\left( \frac{k}{\mu} \right) \text{displaced}}
\]

Mobility Ratio Effects

<table>
<thead>
<tr>
<th>( M )</th>
<th>Neutral</th>
<th>Water and oil move equally well</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \leq 1 )</td>
<td>Favorable</td>
<td>Oil will move easier than water</td>
</tr>
<tr>
<td>( &gt; 1 )</td>
<td>Unfavorable</td>
<td>Water will move easier than oil</td>
</tr>
</tbody>
</table>
Mobility Ratio

- Mobility ratio has a controlling influence on the areal sweep efficiency of a waterflood.
- The mobility of the water must be sufficiently low and that of the oil sufficiently high to provide a reasonably high areal sweep efficiency and thus economically viable improved oil recovery.
- In general, sweep efficiency and oil recovery decrease as mobility ratio increases.

Significance of Mobility Ratio

- Mobility ratio is a key element in the design of a waterflood. It is the principal indicator used to determine sweep efficiency.
- Often waterflooding pattern performance is represented graphically as a function of mobility ratio.
- A mobility ratio greater than unity, $M > 1$, is called an unfavorable mobility ratio.
  - Water can flow through the rock better than oil. The water behind the front moves faster than the oil ahead of the front. As a result, the water does not displace the oil as efficiently as it advances towards the production well.
- A mobility ratio less than unity, $M < 1$, is called a favorable mobility ratio.
  - Oil flows more easily through the formation than water. The water moves more slowly than the oil leading to higher water saturations behind the front. As a result, the water sweeps the oil towards the producer more efficiently resulting in improved oil recovery.
Analyzing Waterflood Patterns

Fluid Displacement in Piston-Like Manner

- Sor
- 1 - Sor
- Water
- Oil
- Distance
- Water Saturation
- Swi

Fluid Displacement in Piston-Like

Buckley-Leverett

Assumptions for basic Buckley-Leverett method:
1. A flood front exists, with only oil moving ahead of the front. Oil and water move behind the front.
2. Reservoir is a single homogeneous layer. Cross-sectional area to flow is constant.
3. Linear steady-state flow occurs and Darcy’s law applies (q injected = q produced), where q is expressed in bd/day.
4. There is no residual gas saturation behind the front.

(5) Fractional flow of the displacing and displaced fluids after breakthrough is assumed to be a function of the mobility ratio of the two fluids (capillary and gravity effects are neglected) as expressed below:

$$f_w = \frac{1}{1 + \left(\frac{\mu_o}{\mu_w}\right)\left(\frac{k_w}{k_o}\right)}$$  (8-6)

(6) Fill-up occurs in all layers prior to flood response. The flood life should be increased to reflect the fill-up period.

Buckley-Leverett

Procedure for basic Buckley-Leverett method:
1. Organize relative permeability data into form suggested in Table 8-8. If several sets of relative permeability data exist for a reservoir, use the set which is representative of the portion of the reservoir to be flooded.

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sw</td>
<td>k_w</td>
<td>k_ow</td>
<td>k_ow/k_w</td>
<td>µ_w/µ_o</td>
<td>f_w</td>
</tr>
</tbody>
</table>

(2) Calculate the fractional flow, f_w, as a function of water saturation, Sw, using equation (8-6) and plot on cartesian coordinate paper as shown in Fig. 8-22. This gives the water saturation value at the flood front at breakthrough. The average saturation behind the front is read at f_w = 1.0.
Analyzing Waterflood Patterns

Buckley-Leverett

(4) Determine graphically the rate of change in the fractional flow, \( f_w' \), as a function of the change in the floodfront water saturation:

\[
\frac{df_w}{dS_w} = \frac{\Delta f_w}{\Delta S_w}
\]

(8-7)

(5) Draw 6 to 8 tangents to the fractional flow curve at \( S_w \) values greater than that at breakthrough. Determine the \( S_w \) and \( f_w' \) values corresponding to these \( S_w \) points.

(6) Plot \( f_w' \) versus \( S_w \) at the flood front on cartesian coordinate paper and draw a smooth curve through the points. Read smooth \( f_w' \) points for each of the \( S_w \) points.

- \[ dx = \left( \frac{q \cdot t}{\phi \cdot A} \right) \cdot \frac{\Delta f_w}{\Delta S_w} \]

(7) Calculate the recovery of oil, \( N_o \), in barrels at breakthrough following the steps in Table 8-9:

\[
N_o = 7705 \cdot A \cdot \phi \left( S_w - S_{wi} \right)
\]

(8-8)

recovery in terms of movable oil

\[
Ed = \frac{\text{avg}(sw)wt - swi}{1 - swi}
\]

(8-9)

(8) Calculate the recovery of oil to each of the \( S_w \) points using equation (8-8) and enter into Table 8-9.

(9) Calculate the water/oil production ratio, WOR, as follows for each of the \( S_w \) points and enter into Table 8-9:

\[
WOR = \frac{B_o}{(1 - f_w)}
\]

(8-9)

(9)

\[
\begin{array}{cccccccc}
S_{wi} & f_w & S_w & S_{wi} - S_{wi} & S_P & f_w' & WOR & t
\end{array}
\]

(10) Calculate the cumulative water injected, \( W_i \), to each of the points as follows and enter into Table 8-9:

\[
W_i = \frac{q \cdot t}{\phi \cdot A}
\]

(8-10)

\[
\frac{df_w}{dS_w} = \frac{\Delta f_w}{\Delta S_w}
\]

(11) Calculate the time, \( t \), to reach each \( S_w \) point as follows and enter into Table 8-9:

\[
t = \frac{W_i}{f_w'}
\]

(8-11)

If the injection rate, \( q \), is not constant throughout the life of the flood, use a time-weighted average rate.

(12) Plot WOR versus \( N_o \) on cartesian coordinate paper. Select a WOR cutoff which is acceptable (90–98%, depending on lifting costs and other expenses).

(13) Plot WOR versus time on cartesian coordinate paper. Determine the life of the flood from the WOR cutoff point.

(14) Plot WOR versus \( W_i \) on cartesian coordinate paper. Determine total water injection from the WOR cutoff point.

\[ E_o = \frac{\text{Volume of oil at start of flood}}{\text{Volume of oil at start of flood}}
\]

(14-7)

\[ E_o = \frac{\text{Pore volume} \cdot \left( S_o - S_{or} \right) - \text{Pore volume} \cdot \left( S_W - S_{or} \right)}{\text{Pore volume} \cdot \left( S_o - S_{or} \right)}
\]

or

\[ E_o = \frac{S_w - S_{or}}{S_W - S_{or}}
\]

(14-7)

where:

- \( S_{or} \) = initial oil saturation at start of flood
- \( B_o \) = oil FVF at start of flood, bbl/STB
- \( S_w \) = average oil saturation in the flood pattern at a particular point during the flood

Assuming a constant oil formation volume factor during the flood life, Equation 14-7 is reduced to:

\[ E_o = \frac{S_w - S_{or}}{S_{or}}
\]

(14-8)
Analyzing Waterflood Patterns

Displacement Efficiency

- Displacement efficiency is defined as the fraction of oil which water will displace in that portion of the reservoir invaded by water.
- This is represented in the figure shown above.
- There are several methods for determining the displacement sweep efficiency.

Waterflood Performance Efficiencies

- Recovery efficiency
  \[ E_R = E_v E_D = E_A E_I E_D \]
  
  \( E_R \) = Recovery efficiency
  \( E_D \) = Displacement efficiency
  \( E_v \) = Volumetric efficiency
  \( E_A \) = Areal efficiency
  \( E_I \) = Vertical efficiency

Calculation of recovery efficiency is difficult because each factor is complex. However, understanding what affects each factor is important to understanding waterflooding.
Performance Efficiencies

- Displacement efficiency ($E_D$)

\[ E_D = \frac{\overline{S}_{wB} - S_{wi}}{1 - S_{wi}} \]

$\overline{S}_{wB}$ is the value of water saturation at the point where the tangent line to the fractional flow curve has a value $f_w = 1.0$. (See Frontal Advance Theory, information from the fractional flow curve.)

Linear Flow Models

- The displacement efficiency of a waterflood is maximized by minimizing the fractional flow of water as a function of water saturation (shift the curve to the right hand side).

Example Displacement Efficiency Calculation

- Given a fractional flow curve, determine the displacement efficiency of the system. $S_{wi}$ is 0.20.
Analyzing Waterflood Patterns

Example
Displacement Efficiency Calculation

Example: Solution
Displacement Efficiency Calculation

Step 3
• Calculate displacement efficiency.

\[ E_D = \frac{S_{\text{wbt}} - S_{\text{wi}}}{1 - S_{\text{wi}}} \]

\[ E_D = \frac{0.72 - 0.20}{1 - 0.20} = 0.65 \]

Example: Solution
Displacement Efficiency Calculation

Step 1
• Draw a tangent to the curve starting from the \( S_{\text{wc}} \) (or \( S_{\text{wi}} \)) value.

Step 2
• Determine the value of the average saturation at breakthrough, \( S_{\text{wbt}} \), at the intersection of the tangent on \( f_w = 1.0 \) line.

Areal Sweep Efficiency (\( E_A \))

Areal Sweep Efficiency (\( E_A \))
Areal Sweep Efficiency

- Defined as the fraction of reservoir area which the injected water contacts.
- The areal sweep efficiency changes with time before and after breakthrough.

Areal Sweep Efficiency (E_A)

- Fraction of the horizontal plane of the reservoir that is behind the flood front at a point in time
- Factors affecting E_A
  - Mobility ratio
  - Well spacing
  - Pattern geometry
  - Areal heterogeneities

Areal Sweep Efficiency
Factors Affecting Areal Sweep Efficiency

- Mobility Ratio
- Dip Angle
- Formation Connectivity
- Fractures
- Areal Permeability Distribution
- Barriers
- Flood Pattern
- Injection Rate
Areal Sweep Efficiency

- Craig’s SPE Monograph 3 contains published design charts and correlations for areal sweep efficiency of a wide range of patterns.

Laboratory experiments have been conducted to determine the areal sweep efficiency of various patterns for different mobility ratios. These results and correlations are discussed at length by Craig in Chapter 5 of the *SPE Monograph Vol. 3*.  

Areal Sweep Efficiency

- Pattern geometry influences areal sweep efficiency
- Correlations exist for common pattern geometries as a function of mobility ratio.
Vertical Sweep Efficiency

- Vertical (invasion) sweep efficiency is defined as the cross-sectional area contacted by injected water divided by the cross-sectional area enclosed in all layers behind the furthest waterflood front.

Vertical Sweep Efficiency

- Vertical sweep efficiency is influenced most significantly by:
  - Mobility ratio
  - Vertical variation of horizontal permeabilities

Vertical Sweep Efficiency

- Water injected into stratified reservoirs will preferentially invade layers of highest permeability.
- The water front will also flow with a greater velocity through these layers.
- The high permeability layers will break through sooner than less permeable layers causing a rapid increase of water cut in the producing well.
- As a result, the economic water cut limit may be reached before less permeable layers have responded to the waterflood.
Mathematical Models of Vertical Sweep Efficiency

- Permeability variation
  - Dykstra and Parsons developed an ideal model that uses a computed coefficient of permeability variation, V. This term is a quantitative indicator of the degree of reservoir heterogeneity.
  - This model: Dykstra Parsons and Stiles methods.

Volumetric Sweep Efficiency

- Volumetric efficiency is defined as the product of the pattern areal sweep efficiency and the vertical sweep efficiency.

\[ E_V = E_A E_I \]  \hspace{1cm} (15)

Vertical Sweep Efficiency

Factors Affecting Vertical Sweep Efficiency

- Vertical Variation of Horizontal Permeability
- Capillary Pressure
- Mobility Ratio
- Injection Rate

Performance predictions Methods

A. Analogy
B. Empirical Techniques
C. Analytical Approaches
D. Material Balance Considerations
E. Simulation Studies
**Analogy**
- In the early stages, prior to sufficient reservoir and production data, analogy is the main method.
- An analogous reservoir in the near-by area can provide a road map provided similarity between the two reservoirs is established or assumed.
- Similarity should be established in reservoir characterization, oil properties, oil-water relative permeability relationship, and pre-flood recovery mechanism.
- Scaling will be required; both PV and OOIP basis are utilized.

---

**Empirical Techniques**
- Many empirical techniques have been proposed in the waterflood literature.
- They are based on the expectation that waterfloods in reservoirs with similar geological and depositional settings would tend to behave similarly.

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**Analytical Techniques**
- **Methodology:** Most of the analytical methods estimate volume of cumulative oil recovery as a function of cumulative water injection.
Material Balance

- MBAL options

Simulation Technique

- The best tool for designing a waterflood project.
- The model is based on integration of all the available static and dynamic data.
- The reliability of the resulting predictions is dependent upon similarity between the reservoir model and the real reservoir.
- The degree of reliability improves if the reservoir simulation model is validated, through the process of history matching, prior to its used as a predictor.