Microwave Network Analysis
Lecture 1: The Scattering Parameters

ELC 305a – Fall 2011

Department of Electronics and Communications Engineering
Faculty of Engineering – Cairo University
Outline

1. **Review on Network Parameters**
   - Impedance and Admittance Matrices
   - ABCD (Transmission) Matrix
   - Example: Transmission Line Section

2. **The Scattering Matrix**
   - Introduction
   - The Scattering Parameters
   - Examples
   - Relation to Other Network Parameters
   - Network Properties
   - Shift in Reference Planes
   - Circuit Analysis
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Impedance and Admittance Matrices

Definitions

\[ V_n = V_n^+ + V_n^- \]
\[ I_n = I_n^+ - I_n^- \]

What is the difference between these matrices and the conventional ones?
Impedance and Admittance Matrices

How to Compute the Matrix Elements

\[ Z_{ij} = \frac{V_i}{I_j} \bigg|_{I_k = 0 \forall k \neq j} \]

The impedance and admittance matrices are the inverses of each other.

\[ Y_{ij} = \frac{I_i}{V_j} \bigg|_{V_k = 0 \forall k \neq j} \]
Impedance and Admittance Matrices

Network Properties

**Symmetric**

\[ Z_{ii} = Z_{jj} \]
\[ Y_{ii} = Y_{jj} \]

**Lossless**

\[ \text{Re}\{Z_{ij}\} = 0 \]
\[ \text{Re}\{Y_{ij}\} = 0 \]

**Reciprocal**

\[ Z_{ij} = Z_{ji} \]
\[ Y_{ij} = Y_{ji} \]

How to prove the losslessness condition?
Two-Port Transmission Parameters

How to compute the ABCD parameters?
What is the advantage of using the ABCD matrix?
How is the ABCD matrix related to the Z (or Y) matrix?
What are the conditions for reciprocity, symmetry, and losslessness?
Similar relations can be derived for the admittance parameters.
Determine the impedance, admittance, and transmission parameters for the lossless transmission line section shown.

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} =
\begin{bmatrix}
\cos \beta \ell & jZ_0 \sin \beta \ell \\
\imath Y_0 \sin \beta \ell & \cos \beta \ell
\end{bmatrix}
\]
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Introduction

What are the Scattering Parameters?

A set of reflection and transmission coefficients defined at the reference planes of the network ports. They relate the incident and outgoing “waves” at the network ports under arbitrary loading/excitation conditions.
Introduction

The Wave Amplitudes and Power

**Incident/Reflected Power on/from Port j**

- **Incident Wave Amplitude**
  
  \[
  a_j = \frac{V_j^+}{\sqrt{Z_{0j}}} = I_j^+ \sqrt{Z_{0j}} 
  \]

- **Reflected Wave Amplitude**
  
  \[
  b_j = \frac{V_j^-}{\sqrt{Z_{0j}}} = I_j^- \sqrt{Z_{0j}} 
  \]

- **Power Calculation**
  
  \[
  P_j^+ = \frac{1}{2} |a_j|^2 
  \]

  \[
  P_j^- = \frac{1}{2} |b_j|^2 
  \]

**How to compute the total power absorbed by the network?**

\[
V_j = V_j^+ + V_j^- = \sqrt{Z_{0j}} (a_j + b_j)
\]

\[
I_j = I_j^+ - I_j^- = \frac{1}{\sqrt{Z_{0j}}} (a_j - b_j)
\]

\[
a_j = \frac{1}{2\sqrt{Z_{0j}}} (V_j + I_j Z_{0j})
\]

\[
b_j = \frac{1}{2\sqrt{Z_{0j}}} (V_j - I_j Z_{0j})
\]

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The Scattering Parameters

The Incident and Reflected Wave Amplitudes

**Reflection Coefficient of Source i**

\[ a_i = \Gamma_{si} b_i + a_{si} \]

**Incident Wave on Port i**

\[ b_i = S_{i1} a_1 + S_{i2} a_2 + \cdots + S_{ij} a_j + \cdots + S_{in} a_n \]

**Outgoing Wave from Port i**

\[ T_{ij} = \frac{b_i}{a_j} \]

**Wave Generated by Source i**

\[ \Gamma_i = \frac{b_i}{a_i} \]

**Transmission Coefficient (from Port j to Port i)**

\[ \Gamma_i = \frac{b_i}{a_i} \]

**Reflection Coefficient (at Port i)**

What is the difference between \( S_{ij} \) and \( T_{ij} \), \( S_{ii} \) and \( \Gamma_i \)?
Review on Network Parameters

The Scattering Matrix

Evaluation of the Scattering Parameters

\[ S_{ij} = \frac{b_i}{a_j} \left| \begin{array}{c} a_k = 0 \forall k \neq j \end{array} \right. \]

\[ S_{ij} = \frac{V_i^- \sqrt{Z_{0j}}}{V_j^+ \sqrt{Z_{0i}}} \left| \begin{array}{c} a_k = 0 \forall k \neq j \end{array} \right. \]

\[ S_{ij} = \frac{\sqrt{Z_{0j}}}{\sqrt{Z_{0i}}} \frac{V_i - I_j Z_{0i}}{V_j + I_j Z_{0j}} \left| \begin{array}{c} a_k = 0 \forall k \neq j \end{array} \right. \]

To evaluate the elements of any column of the scattering matrix, a source should be connected to the corresponding port and all the other ports should be terminated in matched loads (why?).
Scattering Matrix for a TL Section

$$S = \begin{bmatrix} 0 & e^{-j\beta \ell} \\ e^{-j\beta \ell} & 0 \end{bmatrix}$$
Examples

Scattering Matrix for a T- and Π-Network
Examples

Different Port Characteristic Impedances
**Transformation from S to Z Matrix**

\[
\begin{align*}
V &= Z_0^{-\frac{1}{2}} V \\
i &= Z_0^{\frac{1}{2}} I
\end{align*}
\]

\[
\begin{align*}
v_n &= V_n / \sqrt{Z_{0n}} \\
i_n &= I_n \sqrt{Z_{0n}}
\end{align*}
\]

\[
a_n = \frac{1}{2} (v_n + i_n) \\
b_n = \frac{1}{2} (v_n - i_n)
\]

\[
Z_0 = \text{diag}\{Z_{01}, Z_{02}, \ldots, Z_{0N}\}
\]

\[
V = Z I
\]

\[
v = Z_0^{-\frac{1}{2}} Z_0^{-\frac{1}{2}} i = Z i
\]

\[
a + b = z (a - b)
\]

\[
b = \left( z + U \right)^{-1} \left( z - U \right) a
\]

\[
S = \left( z + U \right)^{-1} \left( z - U \right)
\]
Network Properties

Reciprocity, Losslessness, and Matching

Reciprocal

\[ S = S^T \]

Lossless

\[ S^{-1} = \left\{ S^* \right\}^T \]

Internal Matching

\[ S_{ii} = 0 \]

How to prove the losslessness condition?
**Transformation Matrix**

\[
P = \text{diag} \left\{ e^{-j\theta_1}, e^{-j\theta_2}, \ldots, e^{-j\theta_N} \right\}, \quad \theta_n = \beta_n l_n
\]

**The Scattering Matrix**

**Shift in Reference Plane**

\[
\text{S}' = \text{PSP}
\]
Analysis of a Two-Port Network Using the Z Parameters

\[ V_1 = Z_{11}I_1 + Z_{12}I_2 \]
\[ V_2 = Z_{21}I_1 + Z_{22}I_2 \]
\[ V_2 = -I_2Z_L \]

\[ Z_{in} = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + Z_L} \]
Circuit Analysis

Analysis of a Two-Port Network Using the S Parameters

\[ \Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \]

\[ b_1 = S_{11} a_1 + S_{12} a_2 \]

\[ b_2 = S_{21} a_1 + S_{22} a_2 \]

\[ a_2 = \Gamma_L b_2 \]
Conclusion

- The scattering parameters and their use in the analysis of microwave circuits.

- The relation between the S matrix and the other network parameters.

- Determining the network properties (reciprocity, symmetry, losslessness, and matching) from the S matrix.

- The effect of shifting the reference plane on the S matrix.

- Circuit analysis using the scattering parameters.